

The Responsiveness of Federal Personal Income Taxes to Income Change

RELIABLE estimates of Federal tax revenues are essential for the evaluation of the economic impact of the Federal budget. One of the purposes of this article is to aid in the preparation of such estimates through the presentation of tax functions that relate Federal personal income tax payments (less tax refunds) to "tax policy variables"—such as the tax and exemption rates—and to population and personal income. The equations are based on annual data for 1947-65 and are developed within the framework of the national income accounts (NIA).¹ These equations can be used, for example, to help estimate tax receipts under the 1965 personal income tax schedule. They can also be used to provide estimates under other schedules.

Another purpose of this article is to present summary measures of the automatic responsiveness of Federal personal income taxes to changes in personal income under the 1965 tax schedule and to compare these with estimates under the 1954 and earlier postwar tax schedules. The summary measures are: (1) The *marginal tax rate*, which shows the absolute change in tax payments per dollar change in income; and (2) the *tax elasticity*, which shows the percent change in tax payments for a 1 percent change in income. (The two summary measures are mathematically related; the tax elasticity is the marginal tax rate divided by the ratio of tax payments to personal income.) The marginal tax rate may be used as an index to compare postwar changes in the automatic stabilizing effect of Federal personal

income taxes on real output and on price change induced by a change in demand. In conjunction with the level of taxes, the tax elasticity may be used to determine whether the tax is an automatic fiscal stabilizer of real output and prices during periods of inflationary changes in demand or cost-push inflation. This condensed statement of the significance of the two summary measures is elaborated in later sections of this article.

Major findings

The primary objective of the Federal personal income tax cuts in 1964 and 1965 was to reduce the restrictive effect of these taxes on the level of demand and output. It was also considered

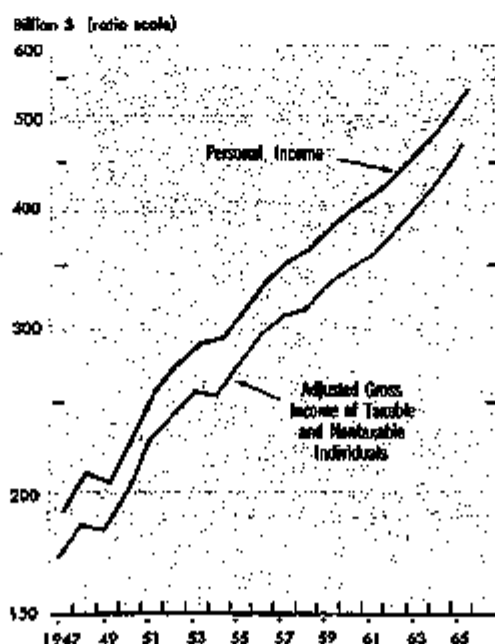
desirable, to the extent possible under the new schedule, to retain their effectiveness as an automatic fiscal stabilizer on output with respect to changes in demand.²

One of the major findings of this article is that this result was essentially realized. The average rate of taxation was reduced significantly, but the marginal tax rate with respect to personal income in 1965 under the 1965 tax schedule—14.5 percent—was only slightly lower than under the 1954 schedule—15.0 percent. (These estimates of the aggregate marginal rate reflect changes in the level and distribution of income as well as changes in the rate structure.) The study also finds that the marginal tax rate under the 1965 tax schedule is positively related to personal income, but the relationship is very weak.

A second major finding is that the tax elasticity with respect to personal income in 1965 under the 1965 schedule, 1.55, was larger than in 1963 under the 1954 tax schedule, 1.41. When these changes are considered in conjunction with the level of taxes in the 2 years, the automatic price-stabilizing effect of Federal personal income taxes was found to be about the same in 1965 as in 1963. The study also finds that the tax elasticity is inversely related to personal income; i.e., the tax elasticity tends to decline as income grows over time.

Finally, on the basis of a limited test it appears that the tax functions predicted extremely well. For 1966, a year not included in the regression analysis, the predicted value of Federal personal income tax payments (less tax refunds) was \$57.0 billion.

CHART 7
Personal and Adjusted Gross Income of Taxable and Nontaxable Individuals



U.S. Department of Commerce, Office of Business Economics

67-12-7

1. This article is taken from a larger econometric study of long-run Federal tax functions done within the NIA framework and undertaken for the Interagency Economic Growth Project.

2. *Economic Report of the President, January 1963*, pp. 68-69.

This was \$1.6 billion below the actual figure of \$58.6 billion, but most of the difference—perhaps as much as \$1.5 billion—can be accounted for by the introduction of graduated withholding rates in 1966, which the equation could not be expected to predict. The schedule of graduated withholding rates should not affect predictions after 1966 when it becomes a normal part of the personal income tax system.

A qualification

A general qualification should be stated at this point. The estimates of the marginal tax rate and the tax elasticity are based on annual data. For questions of short-run stability, it would be more useful if the summary measures were based on quarterly or even monthly data. Estimates based on annual data tend to be somewhat larger than those based on quarterly data because on a quarterly basis nearly all of the automatic response to changes in current income is limited to the withheld portion of the taxes.³ However, this does not affect the general conclusions based on comparisons under the 1965 and earlier postwar tax schedules. Also, for post-1966 analyses, the quarterly and annual estimates should be closer because of the introduction of graduated withholding rates.

The remaining sections in the article are as follows: The first section briefly reviews postwar trends in the basic series used in the article. The second section provides a discussion of the summary measures or tax parameters

and their interpretation in the article. The third section presents the estimated tax functions. The fourth and final section presents the estimated values of the summary measures and discusses their implications.

Postwar Trends in Factors Affecting Taxes

There is a considerable difference between personal income and "adjusted gross income," the gross income concept used for income tax calculations. According to OBE estimates, total adjusted gross income (AGI) of taxable and nontaxable individuals was \$468.7 billion in 1965, \$69.1 billion less than personal income (table 1).⁴ The portion of personal income not included in total AGI amounted to \$100.6 billion; the major items excluded from AGI were transfer payments, other labor income, and imputed income. On the other side of the ledger, \$31.5 billion included in total AGI was not in personal income. The principal items were contributions of employees and self-employed persons for social insurance and net gains from the sale of capital assets.

Total AGI exhibited about the same annual movements as personal income from 1947 to 1965 (chart 7). However, there was a slight downward shift in the level of total AGI relative to personal income beginning in 1958. The ratio of total AGI to personal income fluctuated within the narrow range of 88 to 90 percent from 1947 to 1957 and between 86 and 87 percent from 1958 to 1965. This shift was due partly to an increase in transfer payments, which are included in personal income but not in total AGI.

Taxable income

Taxable income of individuals (AGI of taxable individuals minus their personal exemptions and deductions) was \$254 billion in 1965 or considerably less than AGI of taxable and nontaxable individuals (table 2). Taxable income as a percent of total AGI increased from 44 percent in 1947 to 54 percent in 1965 (chart 8). This reflects a rise in the proportion of total AGI reported by taxable individuals and a rise in

4. The annual series on total AGI and its reconciliation with personal income for 1947-65 presented in this paper is an extension and revision of previous estimates made by OBE. "The Tax Base for Individual Income," *SURVEY OF CURRENT BUSINESS*, May 1963, p. 3. The method used to construct the series on total AGI was originally developed by Joseph A. Pechman, "Yield of the Individual Income Tax During a Recession," *Policies to Combat Depression*, Conference of Universities—National Bureau Committee for Economic Research (Princeton University Press, 1966), p. 143.

3. Albert Ando, E. Cary Brown, Robert M. Solow, and John Kareken, "Lags in Fiscal and Monetary Policy," *Stabilization Policies* (Commission on Money and Credit and Prentice-Hall, 1962), pp. 97-102.

Table 1.—Reconciliation Between Personal Income and Adjusted Gross Income of Taxable and Nontaxable Individuals, 1947-65

(Billions of dollars)

	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
1. Personal income.....	191.3	218.2	207.2	227.5	256.6	272.6	299.2	290.1	310.9	333.0	351.1	381.2	383.5	401.9	406.6	422.6	455.5	497.5	537.8
2. Deduct: Portion of personal income not included in adjusted gross income.....	25.2	31.1	29.9	33.9	38.0	40.1	41.4	40.7	49.3	51.9	53.0	65.1	68.2	72.7	78.1	82.0	87.6	92.1	100.6
(a) Transfer payments (except military retirement pay).....	11.6	11.0	12.1	14.9	19.2	12.7	13.6	13.6	15.9	18.0	20.9	25.1	26.0	27.8	31.6	32.4	34.3	35.5	38.4
(b) Other labor income (except directors' fees).....	2.3	2.0	2.9	3.7	4.7	6.2	8.0	6.2	7.3	8.2	9.3	9.7	11.1	11.8	12.6	12.6	14.6	16.4	18.3
(c) Imputed income.....	8.7	9.8	9.5	10.7	12.5	14.0	15.3	16.2	16.8	17.5	18.4	19.6	20.7	21.9	22.7	22.6	23.6	27.7	29.8
(d) Other types of personal income.....	2.6	3.2	5.4	4.6	7.2	8.2	6.6	3.7	3.4	8.1	0.4	10.7	10.4	11.2	11.3	12.4	12.1	13.5	14.2
3. Add: Portion of adjusted gross income not included in personal income.....	5.8	6.2	6.0	7.9	8.9	8.5	8.4	10.3	12.5	13.6	14.1	15.6	18.4	18.1	21.2	20.4	22.2	27.2	31.5
(a) Employees and self-employed contributions for social insurance.....	2.1	2.2	2.2	2.9	3.4	3.8	4.0	4.6	5.2	5.8	6.7	6.9	7.9	9.3	9.6	10.3	11.8	12.5	15.4
(b) Net gains from sale of capital assets.....	2.2	2.2	1.6	2.9	3.0	2.5	2.1	3.4	4.8	4.6	2.5	4.3	6.3	4.3	7.6	3.8	6.4	7.9	10.0
(c) Other types of income.....	1.5	1.8	2.2	2.1	2.5	2.2	2.3	2.3	2.5	3.0	3.0	4.4	4.2	2.5	4.0	4.3	5.0	6.8	8.1
4. Equals: Total adjustments for conceptual differences (2-3).....	19.4	24.9	23.9	26.0	27.7	31.6	33.0	36.4	36.8	38.5	43.0	49.5	49.8	54.6	56.9	61.6	64.4	64.9	69.1
5. Estimated adjusted gross income of taxable and nontaxable individuals.....	171.9	193.3	183.3	201.5	227.9	240.9	255.2	253.7	274.1	294.5	307.2	311.7	333.7	346.4	355.9	381.0	401.1	432.6	468.7

NOTE.—A more detailed reconciliation is available on request.

SOURCE: U.S. Department of Commerce, Office of Business Economics.

the proportion of their AGI that was taxable income.

The postwar rise in taxable income relative to AGI reported on taxable returns was the result of a decrease in the relative importance of personal exemptions, which was only partially offset by an increase in the relative importance of personal deductions. The drop in 1948 was the result of an increase in the personal exemption rate from \$500 to \$600 under the Revenue Act of 1948. The ratio of personal exemptions to AGI of taxable individuals declined from 38 percent in 1948 to 23 percent in 1965. Over this period, deductions as a fraction of AGI of taxable individuals increased from 12 to 16 percent; almost 90 percent of this increase was in itemized deductions.

Tax liabilities

Under the Revenue Act of 1954, which was in effect from 1954 to 1963, individual income tax liabilities (after credits) increased at the same rate as taxable income so that the ratio—the average effective tax rate—remained virtually constant at 23 percent (chart 9). This is surprising since the individual income tax is progressive and since taxable income per taxable return rose about 50 percent over the period. The major explanation lies in the wide tax brackets that were used in the 1954 tax schedule. For individuals filing joint returns, which account for the bulk of taxable income, the upper limit of the lowest tax bracket was \$4,000. A married taxpayer with two dependent children using standard deductions could have doubled his AGI from 1954 to 1963, reported a figure of \$7,100 in 1963, and still have remained within the initial tax bracket. Also, increases in the average rate resulting from the movements of individuals into higher tax brackets were apparently offset by the lower tax rate of previously nontaxable individuals entering the initial tax bracket. Similar comparisons for 1947-53 indicate that the average effective tax rate paralleled the statutory rate for the lowest tax bracket under the earlier postwar tax schedules.

The progressivity in the 1965 tax schedule can be expected to have a more noticeable effect on the average effective tax rate since the initial class in the earlier schedules was divided into four classes in the 1965 schedule. To continue the example cited above—the taxpayer who doubled his AGI and was still taxed at the same initial-bracket rate under the 1954 schedule would have experienced three rate changes under the 1965 schedule.

Federal personal income tax payments

NIA Federal personal income tax payments, the major interest of this article, and individual income tax liabilities as reported in *Statistics of Income* differ in both scope and timing. The

NIA series includes payments under the fiduciary income tax and collections from IRS audits, which are not reflected in *Statistics of Income*. The NIA series also measures taxes when they are paid; *Statistics of Income* shows liabilities.

For the period 1947-65 as a whole, personal income tax payments (less refunds) were on the average about 3 percent above individual income tax liabilities as reported in *Statistics of Income*.⁵ The two series moved in the same direction each year, but the magnitude of the changes was often significantly different (chart 10). In 1964, for example, the NIA series declined \$3.2 billion whereas liabilities declined \$0.9 billion. This large difference occurred mainly because taxes were under-withheld.

Summary Measures

Two aggregate measures, or tax parameters, are generally employed to summarize the automatic responsiveness of taxes to income change—the marginal rate of taxes with respect to income and the elasticity of taxes with respect to income. As defined in this article, the marginal tax rate measures the absolute dollar change in Federal personal income tax payments (less refunds) per dollar change in personal income; the elasticity measures the percent change in these tax payments per 1 percent change in personal income. These summary measures are built up from component parts that are discussed in a later section of the paper. This section is limited to a discussion of the significance of the two overall measures.

Use of summary measures

In this article, a given income tax schedule is viewed as a more effective automatic fiscal stabilizer of real output than an alternative schedule if a change in autonomous demand such as defense expenditures induces a smaller absolute change in real output under the given schedule.⁶ Similarly, a given income tax schedule is viewed as a more effective automatic fiscal stabilizer of prices than an alternative sched-

ule if the change in prices induced by a change in autonomous demand or other factors is smaller under the given schedule. It should be noted that the term "stabilizer" is used in the technical sense of causing real output and prices to converge to finite levels.

The marginal tax rate can be used as an index of the stabilizing effect of the income tax on changes in real output—and on price changes associated with changes in real output—induced by a change in demand. The higher the marginal tax rate, the larger these effects are.

In the case where changes in prices are not associated with changes in real output, the marginal tax rate does not tell us whether the taxes will be stabilizing as defined above. Consider, for example, the extreme case of a full-employment economy where there is an autonomous increase in demand so that only prices and money incomes (but not real incomes) rise. If, in this

5. For example, in 1962 NIA Federal personal income taxes (less refunds) were about \$45.5 billion as compared with \$44.0 billion in individual income tax liabilities (after credits). Total collections from fiduciary income taxes and IRS audits were \$1.3 billion, about 30 percent of the difference between the two series in that year.

6. The discussion in this section follows E. Cary Brown, "The Static Theory of Automatic Fiscal Stabilization," *Journal of Political Economy* (October 1955), pp. 427-490.

pure inflationary situation, we compare two proportional income taxes, the one with the larger marginal rate will moderate prices more than the one with the smaller marginal rate—i.e., the percentage increases in prices will be smaller. But, for reasons explained later, real aggregate demand will not be reduced under either of these proportional taxes, and in this simplified case, the inflation will continue indefinitely. In other words, the proportional income tax in this case will not help to reduce the excess demand, i.e., it will not be stabilizing.

Suppose now we have a price change induced by an increase in autonomous demand when the economy is operating at full employment as in the above example, or suppose the price rise is the result of a cost-push inflation. Under these circumstances, the elasticity of the income tax is pertinent for determining whether the tax helps to stabilize changes in real output and prices by reducing excess demand. If the elasticity of the tax is unity (the proportional tax), the tax is neutral, in the sense that it will not affect real aggregate demand and therefore will not help to stabilize prices. If the elasticity is greater than unity, it has a stabilizing effect in this respect. And if it is less than unity, its effect is destabilizing.⁷

This condensed statement of the significance of the two summary measures is elaborated below, first with reference to the marginal tax rate and then with reference to the tax elasticity.

Marginal tax rate

Assume that a change occurs in autonomous demand—say an increase

in defense expenditures. If resources are not fully employed, this will result in an increase in production, in consumer incomes paid out in the course of production, and hence in consumer demand. This, in turn, will result in further rounds of increases in demand and production which will converge to a finite total—the well-known “multiplier” effect.

7. In order for personal income taxes to affect real disposable income—and thus aggregate demand—changes in money income have to result in changes in real tax payments. More technically, let personal income taxes measured in current prices, T , be a function of current money income, Y —that is, $T = F(Y)$. Given an index of consumer prices, P , and using T^* for real taxes, and Y^* for real income,

$$T^* = \frac{T}{P} = \frac{F(Y)}{P} \text{ and } Y^* = \frac{Y}{P}$$

The change in real taxes under the simplifying assumption that P and Y^* are independent can be written:

$$dT^* = \left[\frac{\partial T^*}{\partial Y^*} \right] dY^* + T^* \left[\frac{\partial T^*}{\partial P} \frac{P}{T^*} - 1 \right] \frac{dP}{P}$$

This equation shows that a change in real tax payments under the simplifying assumption, can be linearly approximated by the sum of two products. The first product comprises the marginal tax rate (measured in constant prices), $\frac{\partial T^*}{\partial Y^*}$, and the change in real income. The second product includes three terms: the tax level, the elasticity of taxes with respect to price, $\frac{\partial T^*}{\partial P} \frac{P}{T^*}$, minus one, and the percent change in prices. That is, a change in real taxes is separated into a real income effect and a price effect and the respective parameters are the marginal tax rate (in constant prices) and the elasticity with respect to price. Note that a change in real income results in a real change in taxes if the marginal tax rate is greater than zero and that an inflationary rise in prices results in an increase in real taxes if the elasticity with respect to price is greater than unity; these are the “critical” values.

In the case of personal income taxes, the tax elasticity with respect to price is also equal to the elasticity with respect to current income. That is,

$$\frac{P}{T} \frac{\partial T}{\partial P} = \frac{P}{T} \left(\frac{\partial T}{\partial Y} \frac{\partial Y}{\partial P} \right) = \frac{P}{T} \frac{\partial T}{\partial Y} Y^* = \frac{Y}{T} \frac{\partial T}{\partial Y}$$

For purposes of this article, the income elasticity—or tax elasticity—was preferred for expository reasons. Thus, after substitution,

$$dT^* = \left[\frac{\partial T^*}{\partial Y^*} \right] dY^* + T^* \left[\frac{\partial T^*}{\partial Y^*} Y^* - 1 \right] \frac{dP}{P}$$

The elasticity of tax yields with respect to price and current income are not necessarily equal in the case of other types of taxes (for example, excise taxes). For a more general analysis that compares different types of taxes as automatic fiscal stabilizers with respect to inflation, see E. Cary Brown, *op. cit.*, pp. 433-439.

The marginal tax rate can be used to gauge the stabilizing effect of the income tax because the higher this rate, the smaller will be the indirect effects of the initial change in demand on real output and prices. This is so because the increase in disposable consumer income—i.e., consumer after-tax income available for further spending—will be lower at each stage that consumer before-tax income is paid out. A higher marginal tax rate will increase the stability not only of real output but also of prices, provided that the price change is positively related to the change in real demand and output.

The moderating effect of income taxes on after-tax income relative to before-tax income can be seen by a numerical illustration for an individual taxpayer. Consider, for example, a taxpayer who is married and has two children. Assume that he experiences an increase of \$3,000 in his before-tax income (AGI), from \$6,000 to \$9,000. If he claims standard deductions (10 percent of his AGI) and \$2,400 in exemptions (\$600 per exemption), his taxable income (AGI minus deductions and exemptions) will increase \$2,700, from \$3,000 to \$5,700. Under the 1935 rate schedule, his taxes will increase \$493, from \$450 to \$943. Thus, his income after taxes will increase \$2,507 as compared with the \$3,000 increase in his before-tax income; the difference is the increase in taxes.

In the example, the marginal tax rate that is analogous to the one estimated below for the economy as a whole is 16.4 percent (\$493/\$3,000)—

Table 2.—Reconciliation Between Adjusted Gross Income of Taxable and Nontaxable Individuals and Taxable Income, 1947-65
(Billions of dollars)

	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
1. Estimate adjusted gross income of taxable and nontaxable individuals.....	171.9	185.3	183.3	201.1	227.9	240.9	255.2	251.7	274.1	294.5	307.2	311.7	333.7	345.4	359.4	361.0	401.1	431.5	463.7
2. Deduct: Nontaxable and nonreported adjusted gross income.....	36.6	43.2	44.7	43.1	44.7	44.3	44.7	44.0	44.5	44.9	45.0	49.5	45.9	49.2	49.0	50.4	50.7	54.6	60.0
3. Equals: Adjusted gross income of taxable individuals.....	135.3	142.1	138.6	158.0	183.2	196.6	210.5	207.7	229.6	249.6	262.2	262.2	287.8	296.2	311.3	310.6	350.4	376.9	403.7
4. Deduct: Deductions of taxable individuals.....	15.6	16.4	16.8	18.0	22.6	24.9	27.3	27.6	30.5	33.6	36.2	37.2	41.7	44.5	47.2	50.5	54.5	58.4	63.0
(a) Standard deductions.....	8.5	9.5	9.1	10.1	11.7	12.2	12.9	11.9	13.0	13.6	12.3	11.7	12.1	11.7	11.6	11.5	11.9	14.8	n.a.
(b) Itemized deductions.....	7.1	6.9	7.7	7.9	10.9	12.7	14.3	15.9	17.5	20.0	23.9	25.5	29.6	32.8	35.6	39.0	42.6	43.6	n.a.
5. Equals: Net income of taxable individuals.....	119.7	125.7	121.8	139.9	160.6	171.7	183.2	180.2	199.1	216.0	226.0	225.0	246.1	251.7	264.1	260.1	295.9	317.6	340.7
6. Deduct: Personal exemptions of taxable individuals.....	44.3	50.9	50.1	55.2	61.4	64.5	68.9	67.0	71.2	74.6	76.9	75.8	79.7	81.2	82.5	85.1	87.4	88.3	91.9
7. Equals: Taxable income of individuals.....	75.4	74.8	71.7	84.7	99.2	107.2	114.3	113.2	127.9	141.4	149.2	149.2	166.4	170.5	181.6	175.0	208.5	229.3	250.8

n.a. Not available.

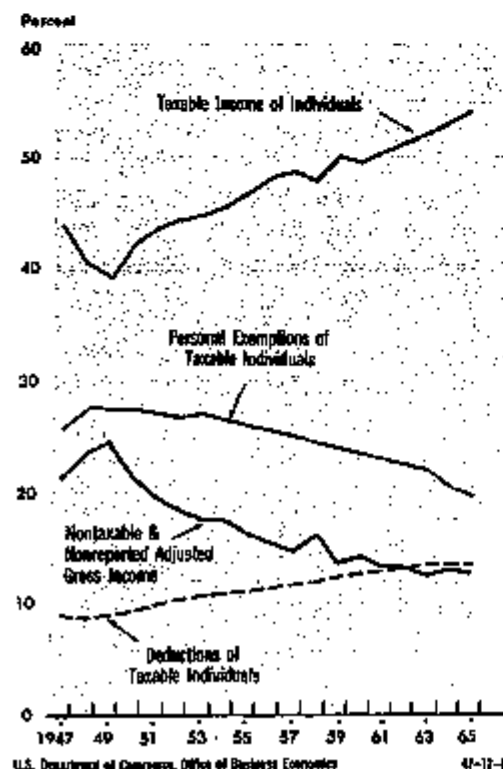
Sources: U.S. Department of Commerce, Office of Business Economics and U.S. Treasury Department.

the ratio of the change in taxes to the change in before-tax income. It shows the extent to which the change in before-tax income was offset by the automatic response of taxes. If, in the hypothetical illustration, there were also no changes in consumer prices, the decrease in after-tax income would represent a decline in the family's real after-tax income. This decline would tend to reduce the family's consumption; however, the reduction would not be as large as it would have been if the tax offset, measured by the marginal tax rate, were smaller. It can easily be seen by reversing the above illustration that the automatic response of personal income taxes also works in the opposite direction; it tends to moderate decreases in after-tax income during economic recessions.⁸

8. The earlier postwar interest in personal income taxes as an automatic fiscal stabilizer centered on its effectiveness during economic recessions. More recent interest has been on its dampening effect or "fiscal drag" during economic recovery and expansion. See, for example, the discussion of the concept of the full-employment budget surplus in the *Economic Report of the President*, January 1962, pp. 77-84.

CHART 8

Components of Adjusted Gross Income of Taxable and Nontaxable Individuals as Percent of Total



Tax elasticity

The tax elasticity is useful in the analysis of a rather different kind of problem. Suppose that we posit a price change—induced either by a change in autonomous demand or by other factors—and wish to trace the effects of this price change on real output and on further changes in prices. If the elasticity of the income tax is unity, the tax is neutral with respect to these further changes. This can be understood as follows: To isolate the effects of a pure price change, let us assume that before-tax incomes and prices increase in the same proportion. On this assumption, real disposable income will be unchanged if the elasticity of the income tax is unity. This is so because before-tax incomes, taxes, and disposable income in current prices will all increase in the same proportion as the increase in prices. There will be no change in real disposable income and no change in real consumer demand, assuming that real consumer demand is a function of real income. Thus, aggregate demand will be unaffected and the pressure on prices will continue.

Now let us compare this situation with one in which the tax elasticity is greater than unity; this is the case of the progressive income tax, the one we are concerned with in this article. In this case, taxes will increase more, and disposable income will increase less than before-tax income in current prices. Since the increase in disposable income in current prices is less than the increase in prices, real disposable income will decline. So will real consumer demand, assuming again that it is a function of real disposable income. Thus, when an increase in before-tax income is simply a reflection of higher prices, an income tax with an elasticity greater than unity leads to a decline in real consumer demand. Accordingly, aggregate demand will be reduced and prices will move toward stability.

Conversely, if the elasticity of the income tax is less than unity, we find that the induced change in prices leads to an increase in real after-tax income and hence real demand. In this sense, an income tax with an elasticity less than unity may be said to have a

destabilizing effect on changes in real output and prices.

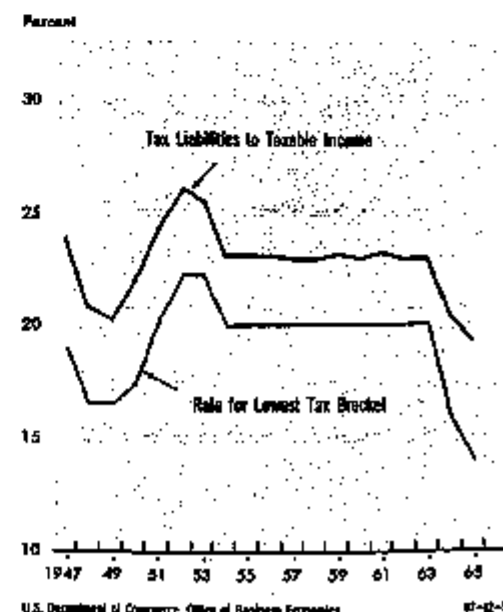
In evaluating the stabilizing or destabilizing effects of tax elasticity, it is necessary to take into account not only the magnitude of the elasticity but also the size of the tax. A small income tax with a very high tax elasticity may have a lesser stabilizing effect than a larger tax with a lower elasticity.⁹

The effect of inflationary increases in before-tax income on real after-tax income can be seen by continuing the previous hypothetical illustration used for the marginal tax rate. Assume that the taxpayer's before-tax income in the first year is \$9,000, that his taxes under the 1965 schedule are \$943, and that this time he experiences a 3 percent increase in both before-tax income and consumer prices in the second year. That is, his money income increases 3 percent, but his real income before taxes is unchanged. In this case, the taxpayer's before-tax income measured in current prices rises \$270, his taxable income rises \$243, and his taxes under the 1965 tax schedule rise \$46. Measured in current dollars, the taxpayer's

9. For a discussion of the problem of using the tax elasticity, see Richard Goode, "The Individual Income Tax," (The Brookings Institution, 1964), pp. 287-288.

CHART 9

Ratio of Tax Liabilities to Taxable Income and Rate for Lowest Income Bracket



after-tax income increases \$224, from \$8,057 to \$8,281. But in constant prices, it decreases \$17, from \$8,057 to \$8,040 (\$8,281/1.03). That is, the taxpayer's after-tax income measured in constant consumer prices decreases \$17 whereas his real income before taxes is unchanged. This decrease in real after-tax income tends to dampen the family's consumption.

The tax elasticity implied in the example that is analogous to the aggregate tax elasticity estimated later in the article is 1.63 (4.9 percent/3 percent)—

the percent change in taxes relative to the percent change in before-tax income. In the above illustration, if the individual's taxes had increased by the same percentage as his before-tax income (tax elasticity of unity), his real income after taxes would have been unchanged.

It is important to note that in order to compute the absolute change in real taxes (\$17 in the example) resulting from the 3 percent inflationary rise in income, it is also necessary to know the level of real taxes in the first year (\$943 in the example).

Econometric Analysis

We now present an econometric analysis in which the NIA annual series on Federal personal income taxes is related to income, policy variables, and other variables. The relation, based on the years 1947-65, is estimated in three stages: The first stage relates taxable income of individuals (as reported in *Statistics of Income*) to personal income; the second stage relates tax liabilities (as reported in *Statistics of Income*) to taxable income; and the third, NIA Federal personal income tax payments to tax liabilities. These equations are discussed in turn. The combined results, including the predictions for 1966, are given in the final subsection.

Taxable income

For questions of fiscal policy and income determination, we are mainly interested in relating tax yields to personal income rather than to AGI. However, in conceptualizing the relation of taxable income to personal income, it is initially useful to view the relation of taxable income to total AGI. (The relation of total AGI to personal income is given by the reconciliation in table 1.) For this reason, the conceptual discussion that follows is almost entirely concerned with relating taxable income to total AGI.

In the case of taxable individuals, the relation of taxable income to AGI is the simple accounting identity used in table 2: taxable income equals AGI minus personal exemptions and per-

sonal deductions. The identity does not hold for all individuals because there are nontaxable individuals whose exemptions and deductions exceed their incomes. Therefore, the relation between taxable income and total AGI has to be formulated in more general terms.

Taxable income is viewed as a function of total AGI, total personal exemptions, total personal deductions, and other factors that describe the distribution of these variables and that are discussed more fully below. Total exemptions and deductions are defined similarly in relation to total AGI as the amount of exemptions and deductions that would be reported if all individuals filed tax returns.

Total exemptions and total deductions are the relevant series, rather than amounts reported in taxable returns, in order to keep the relation between changes in exemptions and deductions and changes in taxable income consistent. This question of consistency can be shown by a simple numerical illustration. Consider a taxpayer whose taxable income is \$500 in the initial year and who, in the second year, experiences no change in AGI or deductions but a \$600 increase in exemptions so that his taxable income drops to zero. Taxable returns (as reported in *Statistics of Income*) would show a \$500 decrease in taxable income and a \$600 decrease in exemptions for the taxpayer himself. Thus, if only those exemptions reported

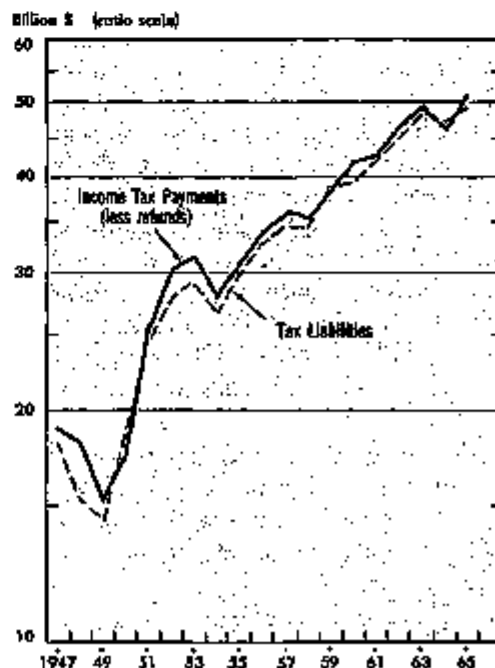
on taxable returns were used, the decrease in taxable income would be associated with a decrease in exemptions. However, if total exemptions of taxable and nontaxable individuals are used, the decrease in taxable income is associated with the increase in exemption—as it should be. The same approach applies to the use of total deductions of taxable and nontaxable individuals rather than those reported on taxable returns.

Growth in population must also be taken into account because it is associated with increases in the total number of taxable and nontaxable individuals and the number of exemptions. Thus, given the level of total AGI, an increase in the total number of individuals would mean lower AGI per individual, which could result in a shift of taxable individuals to the nontaxable category. Also, an increase in population automatically results in an increase in exemptions.

Even with no changes in total AGI, total exemptions, or total deductions, taxable income could change as a result of a redistribution of these factors among individuals. Thus, other

CHART 10

Federal Personal Income Tax Payments and Tax Liabilities



U.S. Department of Commerce, Office of Business Economics

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things being equal, a decrease in AGI of a nontaxable individual coupled with an equal dollar increase in the AGI of a taxable individual results in no change in AGI but in an increase in taxable income. This means that conceptually the general function for taxable income should include statistical measures that describe the joint distribution of individuals with respect to AGI, exemptions, and deductions.

The equation used for the form of the function for taxable income does not assume a constant marginal rate or a constant elasticity for taxable income with respect to AGI. There is reason to believe that they may not be constants. For the taxable individual with given exemptions and constant deductions or deductions approximately proportional to their AGI, the elasticity is inversely related to their AGI.¹⁰ The elasticity of taxable income with respect to AGI tends to decline for the taxable individual because his marginal rate of taxable income with respect to AGI is relatively constant while the ratio of his taxable income to his AGI tends to increase because of fixed exemptions. The elasticity also tends to drop off sharply in the range of incomes less than \$15,000 for taxpayers who have four or fewer exemptions and whose deductions are approximately proportional to their AGI.

The marginal rate of taxable income with respect to AGI was assumed not to be constant because it tends to increase as incomes increase and as individuals become taxable. Similarly, the marginal rate for taxable income will tend to decrease as incomes decrease and as individuals become nontaxable. Thus, in an aggregate equation that includes both taxable and nontaxable individuals, the marginal rate for taxable income with respect to AGI will tend to be positively correlated with AGI.

The form of the equation used for taxable income implies that the elasticity of taxable income with respect to AGI is related inversely to per capita

AGI and that the marginal rate is related positively to per capita AGI. The strength of these relations is determined by the data.

We turn now to the question of relating taxable income to personal income rather than total AGI. The relation of taxable income to personal income can be obtained by simply using personal income and the reconciliation items shown in table 1 instead of total AGI. The approach employed here is to relate taxable income directly to personal income and to test reconciliation items in the function in order to see whether they yield any additional explanation to movements in taxable income. For purposes of comparison, regression results relating taxable income directly to total AGI are also shown in a later footnote.

The equation used to relate taxable income to personal income was chosen on the basis of the above considerations, the manageability of the data, and experiments with alternative forms. It is from Brown and Krutzenka.¹¹ The equation is:

$$(1) \left(1 - \frac{Y_{TI}}{Y_{PI}}\right) = a_0 \left(\frac{Y_{PI}}{N}\right)^{a_1} \left(\frac{E}{N}\right)^{a_2} \dots$$

where

Y_{TI} = taxable income of individuals, billions of dollars,

Y_{PI} = personal income, billions of dollars,

E = total personal exemptions, billions of dollars,

N = total population, billions.

In order to simplify the presentation, only empirically significant variables are shown. The dots at the end of the equation indicate that the other variables conceptually considered above, such as personal deductions, were also included. The empirical results relevant to these other variables are briefly reviewed at the end of the discussion on taxable income.

Equation (1) relates 1 minus taxable personal income as a proportion of total personal income to per capita personal income and to per capita total

personal exemptions. The dependent variable, 1 minus the proportion of taxable income to personal income, is used in order to fix an upper limit of unity on the ratio of taxable income to personal income. If this were not done, projections might yield ratios showing taxable income greater than personal income. Total exemptions measure the dollar amount of exemptions for all individuals, taxable and nontaxable. Average exemptions reflect changes in the statutory exemption rate and, beginning in 1948, changes in the relative importance of persons 65 years of age and older.¹²

The coefficient with respect to per capita personal income, a_1 , measures the percent change in the proportion of nontaxable personal income per 1 percent change in average personal income. It should be negative because an increase in average income decreases the percentage of nontaxable personal income or, stated in terms of its complement, increases the percentage of taxable income. The coefficient with respect to per capita total exemptions, a_2 , measures the percent change in nontaxable personal income per 1 percent change in average exemptions. It should be positive because an increase in average exemptions decreases the percentage of taxable income.

It was found that the results could be improved by including a "dummy" variable for 1958-65 in order to take account of the previously noted shift in total AGI relative to personal income. The estimate of equation (2) based on annual data for 1947-65¹³ is:

12. The series for total exemptions was constructed as follows: For 1947, total population was multiplied by \$300; beginning in 1948, population under 65 was multiplied by \$300 and population 65 and over was multiplied by \$1,200 to take account of their eligibility for double exemptions. The series does not take account of double exemptions for blind persons, or children under 19 years old and students who earn income but receive more than one-half of their support from their parents.

13. The statistical results in logarithmic form are:
 $\log \left(1 - \frac{Y_{TI}}{Y_{PI}}\right) = -0.052 - 0.360 \log \left(\frac{Y_{PI}}{N}\right)$
 $+ 0.337 \log \left(\frac{E}{N}\right) + 0.002 D_{58-65} - u$
 $(0.0140) \quad (0.0237) \quad (0.022)$
 $\bar{R}^2 = .987 \quad d = 2.27 \quad \bar{S} = .0028$

\bar{R}^2 is the coefficient of determination adjusted for degrees of freedom; d is the Durbin-Watson test statistic for serial correlation in the residuals; and \bar{S} is the standard error of the equation in logarithmic form adjusted for degrees of freedom. The numbers in parentheses under the estimated coefficients are their respective estimated standard errors.

The regression results are shown in logarithmic form because they were estimated in this form, and some of the test statistics do not apply in the transformed values.

10. Data from the *Statistics of Income* for 1963 show that the ratio of total personal deductions to AGI for taxable individuals varied between about 14 and 18 percent for individuals whose AGI was between \$1,000 and \$100,000.

11. E. Cary Brown and Richard J. Krutzenka, "Income Sensitivity of a Personal Income Tax," *Review of Economics and Statistics* (August 1959), pp. 280-289.

$$(2) \left(1 - \frac{Y_{PI}}{Y_{PI}}\right) =$$

$$.8219(1.0191)^{D_{38-65}} \left(\frac{Y_{PI}}{N}\right)^{-.3369} \left(\frac{E}{N}\right)^{.3707}$$

D_{38-65} is the dummy variable equal to 1 in 1958-65 and zero in all other years. This means that the constant term is equal to 0.8219 for 1947-57, when the dummy variable is zero and 0.8376 ($= .8219 \times 1.0191$) for 1958-65, when the dummy variable is 1.

The fit is very close (chart 11) and there is no significant serial correlation in the residuals. Except for 1959, the differences between the actual and computed values are within about \$2 billion. However, the equation tends to understate declines in taxable income during recessions. The results, incidentally, are almost as good if the dummy variable is omitted.

Other variables tried

Several series measuring per capita personal deductions were also tried, but in each case the estimated coefficient was numerically small and had the wrong sign. Although none of the variants used was the conceptually correct series for total deductions, the results strongly suggest that the effects of changes in deductions on taxable income cannot be statistically separated even if the ideal series were available. The problem is that deductions and income are too closely correlated to estimate their separate effects. A variable describing changes in the relative distribution of AGI was also tried, but its estimated coefficient was small and statistically nonsignificant.

In order to take account of the effects of major reconciliation terms between personal income and total AGI, equation (1) was also estimated using personal income minus transfer payments (except military retirement pay) rather than personal income. The results were slightly poorer than when the dummy variable was excluded from equation (2). The use of average capital gains on the sale of capital assets was also tried and it too was not statistically significant. As might be expected, the statistical fit of equation (1) for 1947-65 using total AGI is somewhat better

than when personal income is used.¹⁴

For forecasting purposes, it is obviously preferable to have an equation in which the estimated coefficients are stable over time. Equation (1) was fitted for the period 1929-65 in order to test the long-term stability of the estimated equation. The estimated coefficients with respect to both average income and average exemptions exhibited slight but statistically significant positive trends for the three and one-half decades as a whole.¹⁵ The trends, however, are not significant when the equations are fitted for the postwar period only. That is, the secular increases in the estimated coefficients between 1947-65 are slight and can be ignored for purposes of this article. It might be added that the statistical results for 1929-65—including trend terms in the coefficients—are remarkably good. Indeed, one of the more interesting statistical results presented in this paper is that one comparatively simple equation fits the data so well, particularly when we consider the number of statutory and other changes that have affected taxable income over the 36-year period.

Tax liabilities

Given the amount of total taxable income reported on tax returns, tax liabilities as shown in the *Statistics of Income* are determined by the statutory rate schedule and the distribution of taxable income by tax rate class. The equation used here to estimate tax liabilities is, of necessity, a simplification. It relates total tax liabilities

14. The statistical results based on total AGI, Y_{AGI} , in logarithmic form are:

$$\log \left(1 - \frac{Y_{PI}}{Y_{AGI}}\right) = -.1357 - .3225 \log \left(\frac{Y_{AGI}}{N}\right) + .3498 \log \left(\frac{E}{N}\right)$$

(.0094) (.0312)

$$R^2 = .992 \quad d = 2.33 \quad \bar{S} = .0031$$

15. The estimated equation in logarithmic form for taxable income for 1929-65 based on personal income with a_1 and a_2 as linear functions of time and including a dummy variable, D_{38-65} , for the war years, is:

$$\log \left(1 - \frac{Y_{PI}}{Y_{PI}}\right) = -.1146 - \frac{(.2138 + .0031t)}{(.0133)(.0006)} \log \left(\frac{Y_{PI}}{N}\right) + \frac{(.2103 + .0038t)}{(.0161)(.0007)} \log \left(\frac{E}{N}\right) + .0227 D_{38-65}$$

$R^2 = .996 \quad d = 1.98 \quad \bar{S} = .0023$ (.0048)

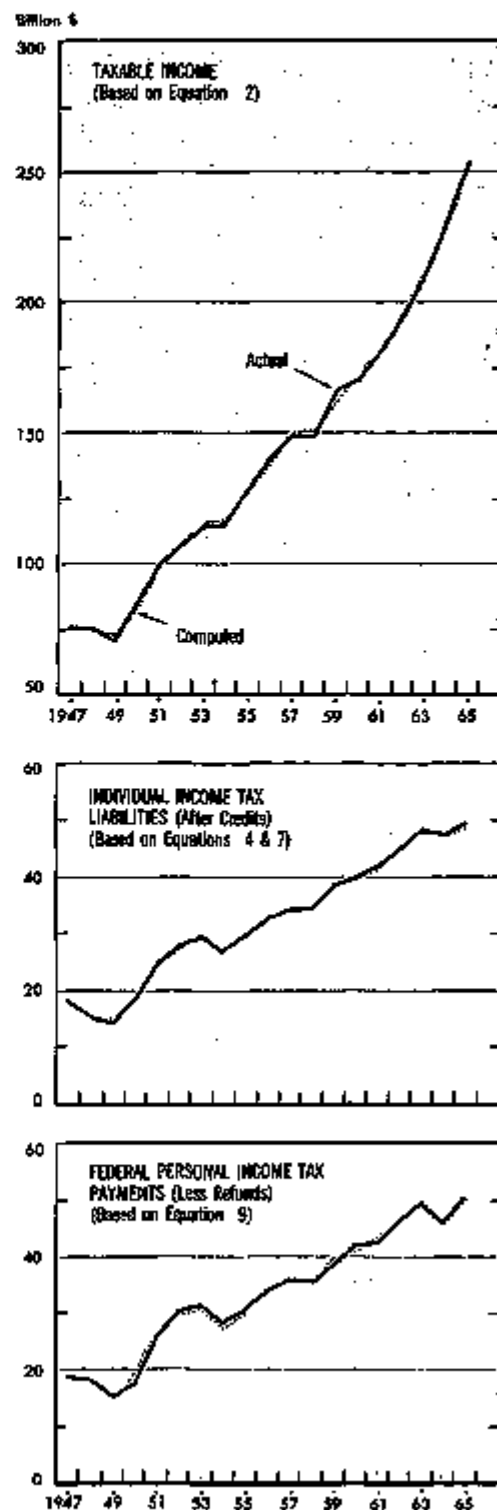
Attempts to explain the trends in the estimated coefficient suggest that they may be due, at least in part, to the omission of personal deductions as an explanatory variable. As has already been noted, even if the ideal series were available, its effect could probably not be statistically separated from the effect of personal income because of multicollinearity. Data for total exemptions for 1929-48 are from Brown and Kruttschnitt, *op cit.*, p. 284, table 1.

to the lowest bracket rate in the tax schedule and to total taxable income. That is,

$$(3) L = b_0 r^{b_1} Y_{TI}^{b_2}$$

CHART 11

Taxable Income, Tax Liabilities and Tax Payments—Actual and Computed



where

L =tax liabilities (after credits),
billions of dollars,

r =lowest bracket rate in the
tax schedule, percent,

Y_{TI} =taxable income, billions of
dollars.

The lowest bracket rate, r (the policy variable), is used to represent the entire statutory rate schedule. The elasticity of total liabilities with respect to the rate, b_1 , is constrained to equal unity so that a given percentage change in the rate results in the same percentage change in liabilities. This is equivalent to multiplying each of the scheduled rates by the same percentage.

The coefficient, b_2 , is the elasticity of tax liabilities with respect to taxable income and shows the percent change in liabilities per 1 percent change in taxable income. It reflects the effect of the progressivity of the tax schedule on tax liabilities. However, it also reflects changes in the number of taxable returns and any other explanatory factors whose changes are correlated with taxable income. The coefficient b_0 represents the effects of all other factors that remained constant during the period studied. The reasons for not including these other determinants in the liabilities equation are explicitly discussed under the statistical results. In the general case when changes in rates are not proportional by tax bracket, new values of b_0 and b_2 must be obtained. One method of obtaining these coefficients is illustrated later by our estimates under the 1965 tax schedule.

Turning to the statistical analysis—there is conceptually a different tax liabilities equation for each of the tax schedules for individual income taxes during the period studied. Nevertheless, it is convenient to combine the 1954 and earlier postwar schedules into one equation and to combine the 1964 and 1965 tax schedules into a second equation. These two equations are discussed in turn.

It was shown earlier (chart 9) that the statutory tax rates were changed every year or two between 1947 and 1953 and that the 1954 schedule was the only one in effect for any number of years, from 1954 to 1963. This means that there was not enough experience

under the earlier tax schedules to estimate both b_0 and b_2 in equation (3).¹⁶ The approach used here is to employ "dummy variables," which separate the effects on liabilities of changes in the statutory tax rates from those that occur automatically as a result of changes in taxable income. This use of dummy variables can be explained more clearly in terms of the actual statistical results.

Equation (3) was fitted to data for 1947-63 employing a dummy variable for each tax schedule except the one in effect during 1948-49. The schedule for 1948-49 was used as the "base" because it yielded the best results. The estimated equation¹⁷ is:

$$(4) \quad L_{TI-43} = 1.2534 r Y_{TI}^{.965} (1.0224)^{D_{48-49}} \\ (1.0212)^{D_{50-53}} (.9741)^{D_{54-55}} \\ (.9482)^{D_{56-58}} (.9432)^{D_{59-63}}$$

The dummy variables, D^i , are equal to 1 for the years shown in the superscript and zero for all other years. They show how the constant term, b_0 , was changed for each of the tax schedules. Thus, for the period 1954-63, when a single tax schedule was in effect, the dummy variable for that period, D_{54-63} , is set equal to 1 and all of the other dummy variables are set equal to zero. Carrying this a step further, the constant term (1.2534) in equation (4) is then multiplied by the base of the dummy variable for 1954 (0.9432), and the product (1.1822) is the estimate of b_0 for the 1954-63 tax schedule. That is, the liabilities equation for 1954-63 reduces to:

$$(5) \quad L_{TI-63} = 1.1822 r Y_{TI}^{.965}$$

The dummy variables for the other tax schedules are similarly interpreted. For the rate schedule in effect in 1948-49, the dummy variables are all equal to zero so that the estimated value of b_0 under this schedule is 1.2534.

16. It should also be noted that income splitting began in 1948 so that the relation between the initial bracket rate and the effective tax rate in 1947 was markedly different than for other years studied.

17. The statistical results in logarithmic form are:

$$\begin{aligned} \log L = \log r + .0081 + .965 \log Y_{TI} \\ (.0090) \\ + .0065 D_{48-49} + .0081 D_{50-53} - .0114 D_{54-55} \\ (.0029) (.0039) (.0031) \\ - .0231 D_{56-58} - .0264 D_{59-63} \\ (.0025) (.0037) \\ R^2 = .999 \quad d = 2.03 \quad S = .0024 \end{aligned}$$

The estimated elasticity of tax liabilities with respect to taxable income is about unity (0.9955). The progressivity of the 1954 schedule, the only one for which there are more than two observations, had no apparent impact on the aggregate relation. As was already noted, there are too few observations under the other schedules to draw any conclusions.

The tax liabilities equation, equation (4), is essentially a compact description of the aggregate relation between tax liabilities and taxable income under the different postwar income tax schedules during the period in which they were in effect. The equation shows the response of liabilities to taxable income under the 1954 tax schedule given the levels and distribution of taxable income that prevailed in 1954-63. It would be inappropriate to use the equation to estimate what liabilities would have been in, for example, 1947-53 if the 1954 tax schedule had been in effect during those years.

In the 1964 and 1965 tax schedules, the previously noted division of the initial tax brackets under the earlier postwar tax schedules into four brackets suggests a likely change in the value of b_0 . That is, the increased progressivity at the lower end of the tax schedule may have a significant effect on the aggregate relation.

There has not been enough experience under the 1965 tax schedule to estimate the two parameters in equation (3) from actual data. Currently, data on liabilities and taxable income are available only for 1965. Joseph A. Pechman has simulated a series of observations under the 1965 tax schedule that can be used together with the actual observation for 1965 to "estimate" the coefficients.¹⁸

The liabilities equation for the 1965 tax rate schedule was estimated in two steps. It was fitted first to Pechman's

18. Pechman simulated (among other things) yearly projections of tax liabilities and taxable income under the 1965 rate schedule. The simulations are for 1965-85 and are based on four assumed growth rates of "ordinary" income (1, 2, 3, and 4 percent) and three assumed exemption rates. Pechman also assumed no change in the relative distribution of income and a constant relation between the number of joint returns and other returns. Pechman's highest assumed rate of growth (4 percent) is lower than the actual rate of growth during 1965 and 1966. See Joseph A. Pechman, "A New Tax Model for Revenue Estimating" (The Brookings Institution, 1965).

simulations, which show yearly observations for tax liabilities and taxable income for different assumed rates of growth. This yielded an estimate of b_1 , the elasticity of tax liabilities with respect to taxable income. Given this estimate of b_1 and its constrained value (unity), the constant term was obtained using the actual 1965 data on liabilities and taxable income, as reported in the *Statistics of Income*. The resulting equation for tax liabilities (after credits) for the 1965 tax schedule¹⁹ is:

$$(6) \quad L_{65} = .6930 r Y_{T1}^{1.1245}$$

Although the fit is very close, there is a good deal of serial correlation in the residuals. Alternative forms were tried, but the results were no better. The very close fit of equation (6) indicates that the form used is at least a very close approximation.

Equation (6) was assumed to apply also to 1964, except for a change in the constant term. That is, it was assumed that the implied elasticity, b_1 , for the 1964 schedule is the same as for the 1965 schedule and that a dummy variable, D_{64} , can be used to adjust b_0 to its 1964 level. The equation pertaining to 1964 is obviously very weak and should be used cautiously. The estimated liabilities equation for both the 1964 and the 1965 schedule is:

$$(7) \quad L_{64-65} = .6930(.9442)^{D_{64}} r Y_{T1}^{1.1245}$$

It should be pointed out that the seemingly close fits (high \bar{R}^2) obtained in this section have to be qualified because of the method used to fit the liabilities equations. The liabilities equation for 1947-63 includes a number of dummy variables in order to take account of changes in income tax schedules

during those years. When a dummy variable is used for a single year, it forces the computed value to equal the actual value for that year. The same is true where the liabilities equation for 1964-65 is applied to 1964 and 1965 because a dummy variable was used for 1964 and the constant term was estimated by equating actual and computed values in 1965.

Federal personal income taxes

NIA Federal personal income taxes measure taxes when they are paid rather than the tax liabilities. The NIA series used here, Federal personal income taxes (less refunds), include withheld and nonwithheld tax payments on current-year liabilities and net yearend settlements, which are the differences between overpayments and refunds on the previous year's liabilities. The difference in timing between the NIA payments series and the *Statistics of Income* liabilities series is reflected in net yearend settlements.

The function for Federal personal income taxes relates tax payments to current-year liabilities and uses a simple hypothesis to explain net yearend settlements. The hypothesis is that taxpayers estimate their quarterly declarations for, say, 1960 on the basis of their liabilities in 1959 and that this essentially determines net yearend settlements in 1961. This hypothesis is incorporated in the equation as the ratio of liabilities in year $t-1$ to liabilities in year $t-2$. A ratio was used in order to avoid statistical problems associated with high intercorrelation among current and lagged values of the liabilities series. The statutory withholding rate can also affect payments for current-year liabilities and net yearend settlements. However, for empirical reasons discussed below, this variable is not included in the equation.

There is no conceptual basis for choosing the form of the equation. The approach here was to choose a form that was simple to estimate and that would not introduce longrun trends in the overall tax parameters.²⁰

20. For example, a linear equation for personal income tax payments, $R = a + bL$, implies an elasticity equal to $\frac{b}{L+b}$.

Since a is negative and b is positive, the elasticity is greater than unity and approaches unity as L increases.

The equation used to estimate Federal personal income tax receipts is:

$$(8) \quad R_t = c_0 L_t^{c_1} \left(\frac{L_{t-1}}{L_{t-2}} \right)^{c_2}$$

where

R = Federal personal income tax payments (less refunds), billions of dollars,

L = *Statistics of Income* individual income tax liabilities (after credits), billions of dollars.

The constant term, c_0 , reflects the difference in scope between NIA Federal personal income tax receipts and *Statistics of Income* tax liabilities, which was discussed in the earlier section on trends. The second term in the equation reflects current-year liabilities. Since withheld taxes and quarterly declarations for current-year liabilities together usually account for nearly all Federal personal income tax payments, the elasticity, c_1 , should be close to unity.

As was discussed above, the final term in the equation is intended to capture net yearend settlements. Its coefficient should be positive. The omission of terms for liabilities before year $t-2$ assumes that net yearend settlements are for liabilities in the immediately preceding year; settlements for earlier years are treated as if they occur randomly.²¹

We now turn to the statistical analysis, where preliminary investigation of the data indicated that the empirical results would be improved by including dummy variables for both 1948 and 1964. In both years, statutory decreases in the withholding rate affected the relation of payments to liabilities.

In estimating equation (8), the elasticity of payments with respect to current liabilities, c_1 , was constrained to equal unity in order to simplify the later analysis of the marginal tax rate. Unconstrained, the estimated value of the elasticity was 0.99.

21. The equation for Federal personal income tax payments (less refunds) might be developed more formally by using the definition of the NIA series. That is, individual equations might be constructed for withheld taxes, for nonwithheld taxes on current-year liabilities and for net yearend settlements; these could then be reduced to a single equation relating payments to current and past liabilities. This approach was rejected because it yields either a complicated nonlinear form or a simple linear form with undesirable implications for the tax parameters. (See footnote 20.)

19. The following are the statistical results in logarithmic form for tax liabilities (before credits) using Pechman's simulated values for 1965-70 to estimate the elasticity of liabilities with respect to taxable income and actual data for 1965 to estimate the constant term:

$$\log L - \log r = -1.593 + 1.1245 \log Y_{T1} \\ (.0033) \\ \bar{R}^2 = .999 \quad d = .98 \quad \bar{F} = .0007$$

The test statistics pertain to the fit of the equation to Pechman's simulated observations—that is, before the adjustment of the constant term to actual 1965 values. The constant term obtained directly from Pechman's simulated observations was 0.6850, which compares closely to the constant term (0.6930), based on actual 1965 data. The constant term was adjusted to actual data for tax liabilities after credits whereas Pechman's simulations are for tax liabilities before credits. Tax credits in 1965 totaled \$0.6 billion, only about 1 percent of liabilities after credits.

The regression equation for Federal personal income tax receipts fitted to data for 1947-65²² is:

$$(9) \quad R_t = 1.0270 L_t \left(\frac{L_{t-1}}{L_{t-2}} \right)^{1.248} (1.1215)_{22}^2 (0.9406)_{22}^2$$

The fit is very close (chart 11)—which is not surprising since current liabilities account for most of NIA payments. The dummy variables are interpreted in the same way as those used previously in the liabilities equation.

The constant term, 1.0270, indicates that the scope of the NIA payments series averaged about 3 percent more than tax liabilities. This is about the expected figure. The estimated coefficient with respect to the final term says that a 10 percent increase in liabilities in year $t-1$ relative to year $t-2$ yields a 1.2 percent rise in payments because of positive net yearend settlements.

The current withholding rate and changes in the withholding rate were also tried, but they were not statistically significant. Further analysis of the data suggests that taxpayers tend to adjust their quarterly declarations to changes in withholdings within the period of a year. For this reason, we expect equation (9) to continue to

apply even after the introduction of graduated withholding rates in 1966.

Combined results

As the introduction pointed out, one of the major purposes of estimating the tax equations is to predict Federal personal income tax receipts directly from personal income, that is, to relate taxes and income within the framework of the national income and product accounts. This means employing equation (2) to estimate taxable income from personal income equation, (4) or (7) to estimate tax liabilities using the estimated taxable income, and equation (9) to estimate Federal personal income taxes (less refunds) using the estimated tax liabilities.²³

Chart 12 shows actual and computed values for Federal personal income taxes for 1947-65. The computed values are based on the three component equations using actual data for personal income, population, and the policy variables. The fit is very close. Except for 1950, the difference is always within \$1 billion. The overall goodness of fit is mainly a test of combining the estimated equations for taxable income (equation 2) and tax payments (equation 9) because of the large number of dummy variables used in fitting the tax liabilities equations.

The tax equations were also used to predict Federal personal income taxes in 1966 on the basis of actual data for personal income, population, and the policy variables. It is an especially difficult year to predict because of the effects of the Revenue Act of 1966. One of the key provisions of the Act was the introduction, in May 1966, of graduated withholding of individual income taxes. The estimated equation for NIA tax payments, equation (9), does not account for this additional contribution to tax payments in that year. Thus, the predicted value of Federal personal income taxes based on the three estimated equations should be below the actual reported figure.

Based on our equations, predicted Federal personal income tax receipts (less refunds) in 1966 was \$57.0 billion, as compared with the actual figure of \$58.6 billion. Of the \$1.6 billion difference, perhaps as much as \$1.5 billion can be accounted for by the introduction of graduated withholding rates. Similar comparisons could not be made for taxable income and tax liabilities because of the actual data for 1966 were not available at the time this article was completed.

Estimated Marginal Tax Rate and Tax Elasticity

In this section, the estimated equations are employed in order to show the automatic responsiveness of Federal personal income tax receipts to changes in income under the 1965 income tax schedule and to compare it with the automatic responsiveness under the 1954 and earlier postwar schedules. As was pointed out in the above section on summary measures, the responsiveness is measured by both the marginal tax rate and the tax elasticity.

The marginal tax rate and the tax elasticity are discussed in turn. They are analyzed in terms of their respective components: the marginal rate

(elasticity) of taxable income with respect to personal income, the marginal rate (elasticity) of liabilities with respect to taxable income, and the marginal rate (elasticity) of tax payments with respect to tax liabilities. The approach is to discuss the annual estimates for the postwar years and to report the underlying equations used to compute these estimates in the footnotes.

Marginal tax rate

The marginal rate for Federal personal income tax payments with respect to personal income, $\frac{\partial R}{\partial Y_P}$, is the product of (1) the marginal rate of taxable income with respect to personal income, $\frac{\partial Y_T}{\partial Y_P}$, (2) the marginal rate of individual

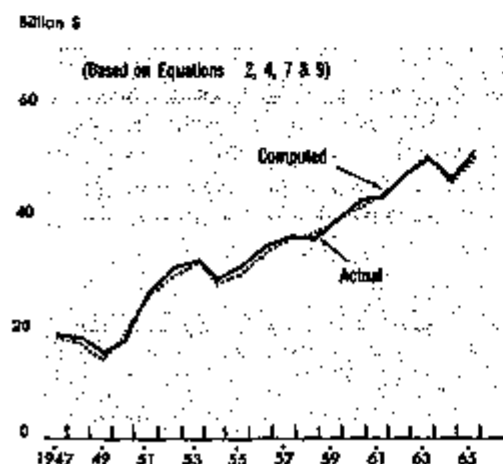
22. The statistical results in logarithmic form are:

$$\begin{aligned} \log R - \log L &= .0018 + .1246 (\log L_{t-1} - \log L_{t-2}) \\ &\quad (.0010) \\ &\quad + .0106 D_{66} - .0206 D_{61} \\ &\quad (.0124) \quad (.0122) \\ R^2 &= .998 \quad d = 2.45 \quad \bar{S} = .0119 \end{aligned}$$

Note: R and L apply to the residuals for $(\log R - \log L)$.

CHART 12

Federal Personal Income Tax Payments Less Refunds—Actual and Computed



23. Alternatively, the three equations might be reduced to one equation by suitable substitution. However, the "reduced form" is extremely cumbersome because of the form of the equation for taxable income.

income tax liabilities with respect to taxable income, $\frac{\partial L}{\partial Y_{PI}}$, and (3) the marginal rate of tax payments with respect to tax liabilities, $\frac{\partial R}{\partial L}$.

$$\text{That is, } \frac{\partial R}{\partial Y_{PI}} = \frac{\partial Y_{PI}}{\partial Y_{PI}} \frac{\partial L}{\partial Y_{PI}} \frac{\partial R}{\partial L}$$

The overall marginal rate of Federal personal income tax receipts with respect to personal income in 1965 was 14.5 percent; a \$1 billion change in personal income resulted in a \$145 million change in Federal personal income tax receipts (column 4, table 3). This was only slightly lower than the marginal tax rate between 1954 and 1963 under the tax schedule; it varied between 14 and 15 percent under the 1954 schedule. The marginal tax rate under the earlier postwar schedules varied between 12 percent (1949) and 16 percent (1952). These schedules and the 1964 schedule were in effect for only 1 or 2 years.

We see that the marginal rate of taxable income with respect to personal income was about 65 percent in 1965 (column 1, table 3). It exhibited the most interesting behavior among the three component marginal rates. The marginal rate for taxable income is positively related to per capita personal income, and as a result, it increased as the economy expanded. However, the added countercyclical effect on consumer after-tax income was small. In the 1948-49 recession, for example, the decreases in the marginal rate from 0.572

to 0.567 resulted in only a \$10 million increase in personal disposable income above what it would have been if the marginal rate had remained constant. It should be added that on an annual basis we cannot get "pure" recession effects because the postwar recessions have been very short.

The positive relation between the marginal rate of taxable income and per capita personal income was not very significant even over the postwar period as a whole. The marginal rate of taxable income with respect to personal income was 8 percentage points larger in 1965 than in 1948, the year the \$600 exemption rate went into effect. The larger marginal rate added less than \$1 billion to Federal personal income tax payments in 1965, based on the marginal rate of tax liabilities and the increase in personal income (\$40 billion) in 1965.

The marginal rate of individual income tax liabilities with respect to taxable income was about 22 percent in 1965 under the 1965 tax schedule (column 2, table 3). It was only a little more than 1 percentage point below the rate under the 1954 tax schedule. The marginal rate of tax liabilities with respect to taxable income was constant—about 23 percent—between 1954 and 1963. The change in the marginal rate for liabilities between 1963 and 1965 reflects the lower rates in the 1965 schedule and an increase in the absolute distribution of taxable income.²⁵

The marginal rate of liabilities with respect to taxable income varied between 20 and 26 percent from 1947 to 1953. But again, it should be underlined that these schedules were in effect only for a year or two and that the experience was too brief to draw any firm conclusions. The same qualification is equally true for the 1964 figure.

25. The difference in the marginal rates for liabilities computed under the 1954 and 1965 tax schedules also reflects the method used to estimate liabilities equation (6) for the 1965 schedule. It will be recalled that the elasticity in the equation was estimated using Peckman's simulated observations under the 1965 schedule and that the constant term obtained from Peckman's data was adjusted to the actual 1965 data. The adjustment of the constant term was, in fact, very small (footnote 10) and within the standard error of the coefficient. The smallness of this adjustment and the extremely good prediction for 1965 using equation (6) indicates that it is probably a good description of reality. At any rate, the likely error because of the method used to estimate equation (6) would probably not affect the general conclusions in the text.

The marginal rate of Federal personal income tax payments with respect to tax liabilities was about the same in 1965 as during 1954-63, except for small year-to-year variations (column 3, table 3). This component generally shows the largest annual movements because it is affected by changes in net yearend settlements. The drop in 1964, for example, reflects the underwithholding of taxes in that year.²⁶

26. It is perhaps useful to employ this 1964 experience in order to illustrate the meaning of the dummy variable with respect to the marginal tax rate and tax elasticity. When the marginal rate for payments with respect to liabilities, including the contribution of the dummy variable, is calculated, it says in effect that the marginal rate was lower in 1964 because of an "unusual" factor—namely, the underwithholding of tax payments. If, on the other hand, we wanted to know what the marginal rate would have been if payments had not been underwithheld, the marginal rate would be computed omitting the contribution of the dummy variable.

When a dummy variable is fitted for only 1 year, the coefficient reflects the random error in the equation for that year. The assumption is made that the error is small compared with the contribution of the unusual factor. This statistical problem does not arise when a dummy variable is used for a period of years as in the equation for taxable income.

Table 3.—Marginal Rates of Federal Personal Income Taxes (Less Refunds) With Respect to Personal Income and Component Marginal Rates, 1947-65

Year	Marginal rates			
	(Percent)			
	(1)	(2)	(3)	(4)
	$\frac{\partial Y_{PI}}{\partial Y_{PI}}$	$\frac{\partial L}{\partial Y_{PI}}$	$\frac{\partial R}{\partial L}$	$\frac{\partial R}{\partial Y_{PI}}$
1947.....	0.508	0.228	1.029	0.145
1948.....	0.572	0.203	1.169	0.136
1949.....	0.567	0.208	1.091	0.116
1950.....	0.579	0.217	1.050	0.128
1951.....	0.592	0.243	1.058	0.123
1952.....	0.593	0.257	1.063	0.134
1953.....	0.593	0.257	1.045	0.129
1954.....	0.602	0.231	1.034	0.143
1955.....	0.602	0.231	1.015	0.142
1956.....	0.615	0.231	1.041	0.145
1957.....	0.618	0.231	1.040	0.145
1958.....	0.618	0.231	1.039	0.146
1959.....	0.619	0.231	1.027	0.147
1960.....	0.622	0.231	1.043	0.150
1961.....	0.625	0.231	1.030	0.148
1962.....	0.630	0.231	1.036	0.151
1963.....	0.635	0.231	1.036	0.151
1964.....	0.641	0.219	0.976	0.137
1965.....	0.648	0.217	1.025	0.145

1. $\frac{\partial Y_{PI}}{\partial Y_{PI}}$ is the marginal rate of taxable income with respect to personal income, based on the equation in footnote 24.

2. $\frac{\partial L}{\partial Y_{PI}}$ is the marginal rate of tax liabilities (after credits) with respect to taxable income, based on the equation in footnote 25.

3. $\frac{\partial R}{\partial L}$ is the marginal rate of Federal personal income tax payments (less refunds) with respect to tax liabilities (after credits), based on the equation in footnote 24.

4. $\frac{\partial R}{\partial Y_{PI}}$ is the marginal rate of Federal personal income tax payments (less refunds) with respect to personal income, based on the product of the three component marginal rates.

24. The equation for the marginal rate of taxable income with respect to personal income implied by equation (2) is:

$$\frac{\partial Y_{PI}}{\partial Y_{PI}} = 1 - .5467(1.0191)^{P_{PI}-49} \left(\frac{Y_{PI}}{N}\right)^{-.234} \left(\frac{E}{N}\right)^{.077}$$

The equation for the marginal rate for tax liabilities with respect to taxable income differs for each of the tax schedules in effect during the postwar years. The marginal rates implied by equation (6) for the schedules between 1947 and 1963 are summarized by the following equation:

$$\left(\frac{\partial L}{\partial Y_{PI}}\right)_{1947-63} = 1.2220(1.0229)^{P_{PI}-49}(1.0212)^{P_{PI}-54}$$

$$(0.741)^{P_{PI}-55}(0.943)^{P_{PI}-56}(0.923)^{P_{PI}-57}$$

(To simplify the computations, the estimated coefficient with respect to taxable income (.9904) in equation (4) was rounded to unity and the constant term was reduced from 1.2534 to 1.2220 to take account of the difference.) The equation for the marginal rate for tax liabilities with respect to taxable income implied by equation (7) for the 1964 and 1965 schedules is:

$$\left(\frac{\partial L}{\partial Y_{PI}}\right)_{1964-65} = .7708(0.943)^{P_{PI}-64} Y_{PI}^{.1240}$$

The equation for the marginal rate for personal income tax payments with respect to current-year liabilities implied by equation (3) is:

$$\frac{\partial R}{\partial L} = 1.0270(1.1215)^{P_{PI}-49}(0.9406)^{P_{PI}-54} \left(\frac{L_{PI}}{Y_{PI}}\right)^{.150}$$

It was pointed out above that the automatic growth in the marginal rate of taxable income in itself has had only a small impact on the overall marginal tax rate over the postwar years. Under the 1954 schedule, the marginal tax rate was constant despite the substantial growth in taxable income. Under the 1965 schedule, the marginal rate of liabilities is positively related to taxable income (footnote 24) so that we can expect the automatic growth of the overall marginal tax rate to be larger under the 1965 schedule than it was under the 1954 schedule—given the same rate of growth of personal income. The difference, however, will probably be very small.²⁷

The major statistical findings for the marginal tax rate and their implications can now be summarized. The marginal rate of Federal personal income tax receipts with respect to personal income was not greatly different in 1965 under the 1965 tax schedule than it was during 1954-63 under the 1954 schedule. This reflects a small automatic increase in the marginal rate of taxable income with respect to personal income that was offset by an equally small decline in the marginal rate of tax liabilities with respect to taxable income under the 1965 schedule. The overall marginal tax rate is positively related to per capita personal income, somewhat more so under the 1965 schedule than it was under the 1954 schedule. However, the difference between actual tax yields and the yields that would have existed if the marginal tax rate were constant has been and remains small for purposes of fiscal policy. These statistical findings imply that Federal personal income taxes as an automatic fiscal stabilizer are about the same under the 1965 tax schedule as they were under the 1954 schedule.²⁸

Tax elasticity

The elasticity of Federal personal income tax receipts with respect to personal income, $e_{R,Y_{PI}}$, is the product of (1) the elasticity of taxable income with respect to personal income, $e_{Y_{TI},Y_{PI}}$; (2) the elasticity of tax liabilities with respect to taxable income, $e_{L,Y_{TI}}$; and (3) the elasticity of Federal personal income tax receipts with respect to liabilities, $e_{R,L}$.²⁹ That is:

$$e_{R,Y_{PI}} = e_{Y_{TI},Y_{PI}} \cdot e_{L,Y_{TI}} \cdot e_{R,L}$$

The overall elasticity of Federal personal income tax payments with respect to personal income in 1965 was 1.55; that is, a 1 percent change in personal income yielded a 1.55 percent change in personal income taxes (column 4, table 4). In 1963, under the 1954 schedule, the overall tax elasticity was 1.41—14 percentage points below the 1965 figure. This difference reflects two offsetting changes: a rise in the elasticity of tax liabilities with respect to taxable income under the 1965 tax schedule—from 1.00 to 1.13—and an automatic decline in the elasticity of taxable income with respect to personal income, from 1.41 to 1.38. The partial explanation suggested in the econometrics section for the rise in the elasticity for tax liabilities with respect to taxable income was the division of the initial tax bracket in the 1954 schedule into four brackets in the 1965 schedule. At any rate, if the unitary elasticity of tax liabilities with respect to taxable income under the 1954 schedule had also prevailed in 1965, the overall tax elasticity would have been 1.38 instead of 1.55.

The elasticity of taxable income with respect to personal income (column 1,

29. The equation for the elasticity of taxable income with respect to personal income implied by equation (2) is:

$$e_{Y_{TI},Y_{PI}} = \frac{1 - (1.0101)^{10} - 0.0007 \left(\frac{Y_{PI}}{N} \right)^{-1.000} \left(\frac{E}{N} \right)^{.000}}{1 - (1.0101)^{10} - 0.0007 \left(\frac{Y_{PI}}{N} \right)^{-1.000} \left(\frac{E}{N} \right)^{.000}}$$

The elasticity of tax liabilities with respect to taxable income implied by equation (4) for 1947-63 and by equation (7) for 1964-65 are, respectively:

$$\begin{aligned} (e_{L,Y_{TI}})_{1947-63} &= 1.0000 \\ (e_{L,Y_{TI}})_{1964-65} &= 1.1248 \end{aligned}$$

(The estimated value of the elasticity in equation (4) was 0.9955, which was rounded in the text to unity.)

The elasticity of Federal personal income tax receipts with respect to current year liabilities implied by equation (8) is:

$$e_{R,L} = 1.0000$$

table 4) exhibited the most interesting behavior among the components. In fact, except for 1964, all of the postwar movement in the overall tax elasticity resulted from changes in the elasticity for taxable income. The elasticity for taxable income rose from 1.52 in 1947 to 1.61 in 1948, the year that the personal exemption rate was increased, and then declined to 1.38 in 1965. This decline from 1948 was automatic in that it reflected the postwar rise in per capita personal income.

According to our estimates, the elasticity of tax liabilities with respect to taxable income was equal to unity from 1947 to 1963 and, as was already noted, increased to about 1.13 during 1965 (column 3, table 4). The last component, the elasticity of Federal personal income tax payments with respect to tax liabilities, was constrained to equal unity (column 3, table 4) for the reasons given in the section on econometric analysis.

The extrapolations discussed above for the marginal tax rate were not carried out for the tax elasticity because its relation to changes in per

Table 4.—Elasticities of Federal Personal Income Taxes (Less Refunds) With Respect to Personal Income and Component Elasticities, 1947-65

Year	Elasticities			
	(1)	(2)	(3)	(4)
	$e_{Y_{TI},Y_{PI}}$ ¹	$e_{L,Y_{TI}}$ ²	$e_{R,L}$ ³	$e_{R,Y_{PI}}$ ⁴
1947	1.617	1.000	1.000	1.517
1948	1.620	1.000	1.000	1.620
1949	1.620	1.000	1.000	1.620
1950	1.585	1.000	1.000	1.585
1951	1.536	1.000	1.000	1.536
1952	1.515	1.000	1.000	1.515
1953	1.499	1.000	1.000	1.499
1954	1.504	1.000	1.000	1.504
1955	1.483	1.000	1.000	1.483
1956	1.484	1.000	1.000	1.484
1957	1.482	1.000	1.000	1.482
1958	1.483	1.000	1.000	1.483
1959	1.483	1.000	1.000	1.483
1960	1.444	1.000	1.000	1.444
1961	1.437	1.000	1.000	1.437
1962	1.422	1.000	1.000	1.422
1963	1.411	1.000	1.000	1.411
1964	1.598	1.125	1.000	1.670
1965	1.377	1.125	1.000	1.549

- $e_{Y_{TI},Y_{PI}}$ is the elasticity of taxable income with respect to personal income, based on the equation in footnote 29.
- $e_{L,Y_{TI}}$ is the elasticity of tax liabilities (after credits) with respect to taxable income, based on the equation in footnote 23.
- $e_{R,L}$ is the elasticity of Federal personal income tax payments (less refunds) with respect to tax liabilities (after credits), based on the equation in footnote 28.
- $e_{R,Y_{PI}}$ is the elasticity of Federal personal income tax payments (less refunds) with respect to personal income, based on the product of the three component elasticities.

27. If we assume a 6 percent annual rate of growth in personal income, which is more than in 1947-65, and a 1.5 percent annual rate of growth in population on the basis of projections made by the Bureau of the Census, the overall marginal tax rate will grow about 0.3 percentage points per year or 1 percentage point after 5 years.

28. Theoretically, the marginal tax rates used to compare taxes as automatic fiscal stabilizers of real output should be measured in constant prices (see footnote 7). Although the marginal tax rates presented in this article are measured in current prices, they are within rounding errors of those measured in constant consumer prices.

capita personal income was unchanged under the 1965 tax schedule. That is, if the rates of growth in personal income and population continue in the neighborhood of those experienced during 1947-65, the decline in the overall tax elasticity will be about the same as those experienced in the postwar years—roughly 1 percentage point per year.

The major statistical findings for the tax elasticity and their implications can now be summarized. The elasticity of Federal personal income tax receipts with respect to personal income was 1.55 in 1965 under the 1965 tax schedule, as compared with 1.41 in 1963 under the 1954 tax schedule. The elasticity is inversely related to per capita personal income because of the inverse relation between the component elasticity for taxable income and average personal income. As a result, it trended downward between 1948 and 1963 because of the growth in per capita personal income. The overall tax elasticity under the 1965 schedule can be expected to continue to trend downward from its higher 1965 level as per capita personal income grows.

As was pointed out in the section on summary measures, the tax elasticity cannot be used by itself to compare different tax schedules as automatic fiscal stabilizers with respect to price changes; it is also necessary to take account of the level of real tax receipts under the different schedules. These levels reflect the change in the statutory tax rates and the indirect effect of the tax schedules on income and prices. It would require an econometric model of the U.S. economy to separate the change in tax yields resulting from the change in the rate structure at a given level of income from the change in tax yields resulting from the change in income brought about by the new rate structure.

The approach used here is to simply compare 1963, the last year the 1954 schedule was in effect, and 1965; this comparison reflects changes in the level and distribution of income as well as changes in the tax rate structure. The estimate of the tax elasticity for 1963 indicates that a 1 percent inflationary rise in personal income would increase tax yields by 1.41 percent in current prices. Similarly, the estimate of the

tax elasticity for 1965 indicates that a 1 percent inflationary rise in personal income would increase tax yields by 1.55 percent in current prices. In order to obtain the absolute effect on real tax yields, we also need Federal personal income tax payments (less refunds) measured in constant prices for the 2 years. In 1963, tax payments deflated by the implicit price deflator for personal consumption expenditures totaled about \$46 billion; in 1965, they totaled about \$47 billion. From the product of these tax levels, the estimated tax elasticities minus unity, and the percentage increase in prices, we find that a 1 percent inflationary rise in personal income would have resulted in about a \$0.19 billion increase in tax payments in 1963 as compared with \$0.26 billion in 1965. The difference in tax payments and thus in disposable income measured in 1958 consumer prices is less than \$0.1 billion. The comparison indicates that personal income taxes had approximately the same effect in both years.³⁰

³⁰ The calculations discussed in this paragraph follow from the equation shown in footnote 7.